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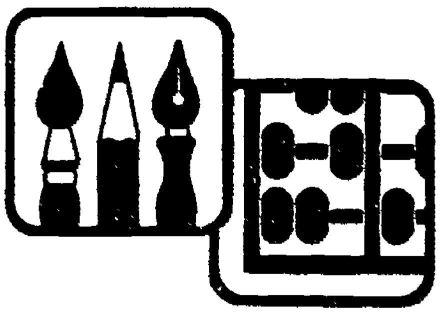
ABSTRACT

The "Clipboard Connection" is a methodology to facilitate the rapid circulation of relevant pre-existing materials from Chapter 1 Technical Assistance Centers of relevant materials from Chapter 1 Technical Assistance Centers (TACs) to their clients, teachers of educationally disadvantaged children in resource centers. Each "Clipboard Connection" consists of a lead sheet summarizing the contents of the materials (reprints of journal articles, brochures, etc.) to be distributed, and the materials themselves. This compilation focuses on techniques for effective mathematics instruction. The following reprints are included: (1) "Teaching Mathematics and Thinking," prepared by Edward A. Silver and Margaret S. Smith (Arithmetic Teacher, volume 37, number 8, April 1990); (2) "Using Writing Activities to Reinforce Mathematics Instruction," by David M. Davison and Daniel L. Pearce (Arithmetic Teacher, volume 35, number 8, April 1990); and (3) "Asking Questions to Evaluate Problem Solving," by Phares G. O'Daffer and Randall I. Charles (Arithmetic Teacher, volume 35, number 5, January 1988). (AF)

The Clipboard CONNECTION

May 1990



Chapter 1
Resource Center
Curriculum & Instruction

MATH+++NOTES

"Teaching Mathematics and Thinking"

Contrary to the argument that basic skills must be mastered first, the authors cite research indicating that "complete mastery of basic skills probably requires" an understanding of when to use a skill, implying that even the use of basic skills involves "thinking, problem solving, and reasoning." Of particular note is the section, *Suggestions for Classroom Practice.*

"Using Writing Activities to Reinforce Mathematics Instruction"

Traditionally, mathematics and writing have not been viewed as interactive disciplines. Davison and Pearce, however, contend that "one of the ways students acquire new information is through putting ideas into language," that writing and mathematics do complement each other, and that content area knowledge acquisition and retention is reinforced through writing activities. In this article, the authors divide writing activities in mathematics into five "categories" and offer specific activites for each.

"Asking Questions to Evaluate Problem Solving"

Another means of promoting analytical skills and their practical application is through the use of questioning. Used as an extension of classroom climate, questioning can stimulate and extend learning. Effective questions encourage risk-taking. They promote discussion and analysis and provide a means by which to evaluate students' thinking and attitudes. In addition to modeling question development, this article also addresses classroom climate, gives a sample problem, and suggests additional readings.

The Clipboard
CONNECTION

May 1990

Chapter 1
Resource Center
Curriculum & Instruction

MATH +++ NOTES

In March of 1989, NCTM published the Curriculum and Evaluation Standards for School Mathematics. Organized into two sections, curriculum and evaluation, these Standards reflect the changes recommended to achieve mathematical literacy in today's society. The Standards support emphasis in three curriculum areas:

problem solving,
communication, and
reasoning.

However, the fourth mathematics assessment (NAEP) indicates that students (and perhaps teachers?) continue to view mathematics as "rule-based and non-creative." These results reinforce the importance of the position taken by NCTM and the Hawkins-Stafford Amendments that all students can benefit from class-room instruction in thinking and advanced skills.

But how will changes in the regular classroom curricula affect Chapter1 programs?

The emphasis in the Hawkins-Stafford Amendments placed on Chapter 1 student success in the regular classroom requires that Chapter 1 professionals be cognizant of such changes. The law's emphasis on "more advanced skills" implies that Chapter 1 educators need to utilize curriculum and teaching strategies that will foster the acquisition and use of higher order skills.

For continuing information about the Standards, read this year's issues of the *Arithmetic Teacher* which carry a regular feature about the Standards and their practical, classroom applications. You can also order a copy of the Curriculum and Evaluation Standards for School Mathematics for $25 from:

National Council of Teachers of Mathematics
1906 Association Drive
Reston, Virginia 22091
(703) 620-9840

RESEARCH INTO PRACTICE

Teaching Mathematics and Thinking

One afternoon, eleven-year-old Michael stopped by the home of an adult friend and found her nearly buried in decorating books, charts, and samples. Michael's sudden appearance at the door was a welcome sight, and he was asked to assist his friend in the process of decorating her home office. Although quite sure he knew little about interior decorating, Michael agreed to lend a hand. The friends began to measure the room and calculate the areas of the ceiling, walls, and floor. They looked at wallpaper swatch books, paint-color charts, and rug samples from various manufacturers and discussed the differences in price, the expected coverage per gallon of paint and roll of wallpaper, and the relative quality and ease of upkeep for different products. One style of wallpaper they liked had a horizontal stripe and required matching. After approximating how many rolls of striped wallpaper they would need to buy, they performed a similar approximation for a design that did not require matching. They looked at carpet samples, discussed the relative merits of light and dark colors and various styles, and considered the issues of cost versus quality with respect to durability and stain resistance. Finally, they selected a set of materials to complete the decorating project based on cost, product quality, maintenance required, and personal preference. After helping his friend with this task for several hours, Michael departed for his home and his dinner.

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In a conversation with Michael the next day, it became clear to the adult friend that Michael saw little relationship between the decorating task and the concept of area as he had studied it in school. In school, he had memorized the appropriate vocabulary and the rules for calculating areas of certain figures, and he had practiced applying the rules to a large number of problems presented in the textbook and on dittoed sheets. His school experience, however, had not revealed a connection between the concept of area and the kinds of interesting and challenging questions that were being considered in the decorating task.

Research on Teaching Mathematics and Thinking

Unfortunately, Michael's view of mathematics—as a subject not connected to interesting thinking—is all too common. On the basis of their school experience, too many students adopt the view that mathematics is a dry and dusty subject. Results from the fourth mathematics assessment of the National Assessment of Educational Progress (NAEP) indicate that the majority of seventh-grade students view mathematics as rule based and noncreative, and many feel that learning mathematics is mostly memorizing (Kouba et al. 1988). Given these responses, it is not surprising that these students performed so poorly on those NAEP problems involving mathematical reasoning, nonroutine situations, or multiple steps in the solution. Carpenter et al. (1988) argued that the NAEP results demonstrate that a greater emphasis must be placed on helping students become better problem solvers who can communicate and reason about mathematics.

Many would argue that the poor performance of students on mathematics problems requiring more than the routine application of a simple procedure is the direct result of excessive instructional attention to low-level knowledge and rules to be memorized. Students are taught to calculate with numbers but not to think in numerical quantities. This emphasis on basic skills is due to a pervasive educational belief that low-level knowledge and skills, those that require little or no independent thinking and judgment, must be taught and learned before high-level thinking can be developed.

The accumulated mass of evidence from the past few decades of cognitively oriented research on mathematics learning and problem solving challenges this belief. For example, evidence (e.g., Carpenter [1985]) has demonstrated that children who lack certain presumed prerequisite low-level skills are often capable of performing high-level tasks successfully using reasoning and problem solving. Furthermore, the process of acquiring and using skills presumed to be low level is itself quite complex (e.g., Wearne and Hiebert [1988]). In fact, complete mastery of basic skills probably requires an understanding of fundamental concepts sufficient to judge when it is appropriate to use a particular skill (e.g., Schoenfeld [1986]). This research base makes it possible to envision a new mathematics curriculum in which the learning of basic skills is integrated with thinking, problem solving, and reasoning.

Thinking skills

In recent years, reform has been called for in many school subjects, not just mathematics, to make the teaching of thinking a central part of the curriculum. The impetus for this movement comes in large part from two different bodies of research: (*a*) research that has revealed and analyzed poor performance by students on complex tasks and (*b*) research that has documented children's capabilities for complex thinking and reasoning on which current curricula are not building. Although interest in teaching thinking is not new, the current focus on teaching *all* students to think, not just a privileged elite, is especially promising. Several programs (e.g., Nickerson, Perkins, and Smith [1985]) have been developed in recent years to respond to the apparent need to upgrade students' reasoning and problem-solving skills (i.e., high-level thinking skills) in many disciplines.

Despite the widespread interest in teaching thinking, no simple, clear, universally accepted definition of what is meant by "high-level thinking" has emerged. Nevertheless, general agreement is found on certain hallmarks of high-level thinking. Resnick's (1987) review of research and scholarship on learning to think identified several of the features that characterize high-level thinking. In particular, high-level thinking is nonalgorithmic and complex; it often yields multiple solutions; and it involves nuanced judgment, uncertainty, and self-regulation of the thinking process. Considering the redecorating task in light of this description, we see that it presented Michael with an opportunity to engage in high-level thinking because (*a*) although some procedural knowledge was required, *no algorithm* was available that would solve the entire problem and no clear path of action had been laid out (e.g., what do you do after you find the areas?); (*b*) the reasoning about the task was *complex*, including many components (e.g., finding the areas, determining the number of gallons of paint needed, weighing cost against quality); (*c*) the problem had *multiple solutions* rather than a single "right answer"; (*d*) *nuanced judgment* was required (e.g., stain-resistant carpet is probably the better choice even though it costs more money because it will wear better and require less maintenance); (*e*) the reasoning involved *uncertainty* (e.g., how many coats of paint will really be required to cover an existing darker color?); and (*f*) the solution process required *self-regulation* (e.g., no checklist of things to consider was available for determining the materials to purchase). Although the redecorating task might have been approached rather algorithmically by an experienced interior decorator, it served as an opportunity for complex quantitative thinking for Michael and his adult friend.

Open-ended problems stimulate thinking.

General versus specific approaches

Much attention has been given to the development of programs for teaching high-level thinking. *General approaches* to teaching thinking skills, such as the Productive Thinking Program (Covington 1985) and CoRT (de Bono 1985), focus instruction on helping students develop a variety of strategies for planning, managing, and monitoring their own cognitive activity. As Adams (1989) noted, the fundamental assumption underlying these general approaches is that a certain set of processes are common to all high-level thinking, regardless of the domain in which they will be applied. *Content-specific approaches*, in contrast, infuse thinking skills and strategies into the teaching of a particular subject area. In elementary school mathematics, the teaching of heuristic problem-solving strategies, such as drawing a diagram or restating the problem, can be part of a content-specific approach, and evidence suggests that students can benefit from such instruction. For example, Lee (1982) was able to teach fourth graders a set of heuristic strategies, and the students who received the instruction were more successful in using the strategies to solve a variety of mathematics problems than students who had not received the instruction. Putt (1979) found that fifth-grade students could benefit from instruction in heuristics, so that they were able to use many of the strategies effectively, they developed an appropriate vocabulary for discussing the strategies, and they were able to suggest many questions appropriate for understanding the problems they were asked to solve. Heuristics are not synonymous with high-level thinking skills, however, and are sometimes inappropriately presented to students as a memorizable list of steps that must be followed.

Resnick (1987) suggests that although evidence shows that elements of high-level thinking can be taught, little evidence can be cited to support a particular program or approach. Perkins and Salomon (1989) argue that if our goal is to teach skills that can be generalized and transferred to other domains, we need to give students instruction in the principles of both general and content-specific high-level thinking. They claim that domain-specific knowledge without general heuristics is inert and that general heuristics that are detached from a rich domain-specific knowledge base are weak. In the next section, we turn our attention to some ways to teach high-level thinking in elementary school mathematics.

Suggestions for Classroom Practice

Evidence suggests that children are capable of high-level thinking and that they often demonstrate this ability naturally in nonschool settings. Helping students to be more thoughtful about the mathematics they learn in school does not necessarily require nonroutine problems or special materials. Opportunities can be found in the topics commonly taught in elementary school. For example, any division computation exercise involving a remainder can be used to model and solve several different story problems. Therefore, any division exercise can serve as an instructional opportunity for rich thinking and discussion about the application of mathematics to real-world situations and the nature of mathematical models (Silver 1988). Moreover, routine computational procedures can in some ways be a source of high-level thinking and reasoning about mathematics (Lampert 1989). For example, having students explore alternative algorithms for standard operations offers an opportunity for a discussion of why a particular algorithm works and why it might be preferable to others. Such opportunities to emphasize high-level thinking are fairly easy to integrate with standard textbooks and instructional practice.

Problems with multiple methods of solution

Opportunities for students to think about mathematics are often associated with their talking about mathematics with each other and with their teacher. One tactic that some teachers have found effective to stimulate students' thinking and discussion is to present problems that have multiple methods of solution. Consider, for example, the following problem that one teacher recently presented to her seventh-grade class:

> A farmer puts his chickens into cages and finds that he has 2 cages left over if he puts 6 chickens in a cage, but he has 2 chickens left over if he puts 4 chickens in a cage. How many chickens and how many cages does the farmer have?

After a brief time in which the teacher presented the problem and discussed it with the class, the students were asked to solve the problem independently or in small groups, using any method they wished. During the solution period, the teacher circulated around the room, taking note of the various approaches being used by the students, asking clarifying questions, and giving other help to those who needed it. During her travels around the room, the teacher designated some of the students to present their solutions to the class. When the time for work on the solution of the problem was over, the designated students, who had written their solutions on large sheets of butcher paper, presented and explained their solutions to the entire class. The multiple methods—some involving clever counting, some based on a guess-and-test strategy, others with a geometric flavor, and even one that was fairly algebraic in nature—were displayed for subsequent summary and discussion, during which the similarities and differences, unique features, and other characteristics of the solutions were identified and examined by the teacher and her students.

Students talk about their approaches.

Opportunities for students to create and discuss multiple solution methods for interesting mathematical problems constitute invitations to engage in high-level thinking in mathematics class, and they represent important opportunities for teachers to learn about and enhance their students' mathematical thinking and reasoning. Although textbooks do not typically emphasize multiple solution procedures, many textbook problems could be used for this kind of activity.

Open-ended problems

Another technique that some teachers have used to foster opportunities for high-level thinking in mathematics classes is the posing of open-ended problems. Although mathematics textbooks almost always present problems that have a well-specified goal, interesting activities can result from modifying existing problems to make them more open ended. Silver and Adams (1987) have suggested a few ways to use open-ended problems in elementary school classes, and Brown and Walter (1983) have suggested many examples appropriate for older students. Consider the following two problems that illustrate the difference between a typical school mathematics problem and a more open-ended version:

Problem 1

> John has 34 marbles, Bill has 27 marbles, and Mike has 23 marbles. How many marbles do they have together?

Problem 2

> John has 34 marbles, Bill has 27 marbles, and Mike has 23 marbles. Write and solve as many problems as you can using this information.

Problem 1, which is a typical elementary school problem, is not likely to encourage high-level thinking if the students are simply expected to apply a particular algorithm they have learned. However, the open-ended nature of problem 2 invites students to consider alternatives, to analyze a simple situation and relate it to the mathematics they know, and to propose problems that are interesting and complex.

Teachers who have used such activities report that their students often propose many interesting problems that are more challenging than the problems they are typically given to solve. For example, problem 2 would likely lead not only to the question posed in problem 1 and other typical questions, such as, How many more marbles than Bill does John have? It would also lead to more interesting and challenging questions, such as, How many more marbles would they need in order to have as many marbles as Sammy, who has 103 marbles? or How many marbles would Bill need to give Mike in order for them to have the same number of marbles? Moreover, a question like the following could lead to an interesting discussion of the reasonableness of problems and solutions: How many marbles would John need to give Mike in order for them to have the same number of marbles?

Situational problems

Many teachers have also found that it is possible to bring into the mathematics classroom problems like the decorating task with which Michael was helping his friend. For example, consider the following:

> Rebecca has a pocketful of change. She would like to buy a soda, which costs $0.55. How could she pay for the soda so that she would eliminate the most change from her pocket?

This applied situational problem has features of the classes of problems previously discussed. It is open with respect to interpretation. (Is Rebecca buying the soda from a machine? If so, even if she had fifty-five pennies, they would not be useful in this situation. But if Rebecca is at a store, fifty-five pennies is a valid solution.) The problem is also open with respect to solution. (If she is buying the soda from a machine, she might first look for eleven nickels in her pocket to eliminate a maximum number of coins. If she does not have eleven nickels, then using nine nickels and one dime might be her next attempt to solve the problem.) Furthermore, the problem is even open with respect to goal. (Should Rebecca use volume, weight, or number of coins in her pocket as the criterion for the "best" solution?) Problems such as this one give students experience in using mathematics and offer opportunities for high-level thinking.

Conclusion

All the activities we have suggested could be used in classrooms by teachers to encourage the kind of high-level thinking described by Resnick (1987)—thinking that is complex, that is nonalgorithmic, and that involves some judgment and uncertainty. Using such activities will not only help students develop their ability to think at a high level but also help them see that such thinking is valued in the subject area of mathematics and in your classroom.

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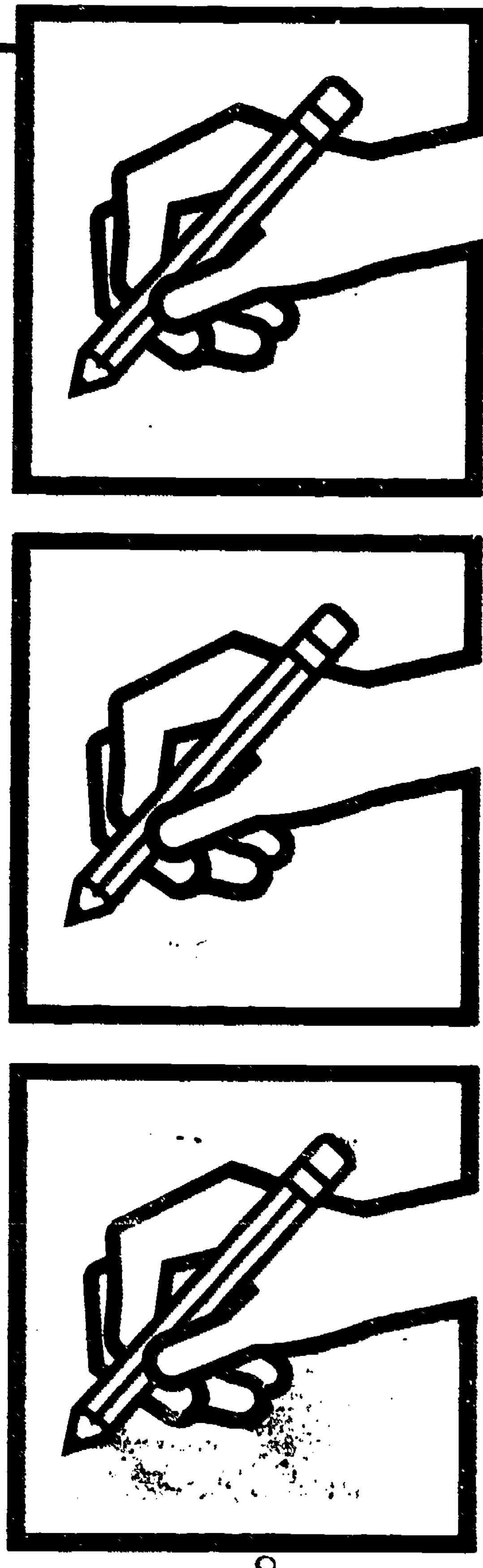
Using Writing Activities to Reinforce Mathematics Instruction

By **David M. Davison** *and* **Daniel L. Pearce**



The traditional view has been that students learn to write in English classes and to compute in mathematics classes and "never the twain shall meet." Certainly little thought has been given to the idea that teachers of various subject areas, especially mathematics, should seek to have students engage in writing activities as part of their study of that area. In recent years, however, this position has been changing, and different authors have recommended increased writing about mathematics by students as a useful and valuable aspect of mathematics instruction (Burton 1985; Greenius 1983; Johnson 1983; Shaw 1983; Watson 1980).

One of the ways students acquire new information is through putting ideas into language. Both Emig (1977) and Haley-James (1982) have stressed that writing is a mode of language that particularly lends itself to the acquisition of new knowledge. Writing about something involves many of the thought processes mathematics teachers would like to foster in their students. Performing a writing task requires students to reflect on, analyze, and synthesize the material being studied in a thoughtful and precise way.

The authors have been exploring various ways that teachers use writing in mathematics instruction. The results of these investigations have suggested that the use of writing activities is sporadic in junior high school mathematics classes (Pearce and Davison, in press). However, a pattern of student writing activities has emerged. In particular, it appears that writing activities used in mathematics classrooms can be classified into five categories: direct use of language; linguistic translation; summarizing; applied use of language; and creative use of language. Writing activities in each of these categories have a use in the mathematics classroom. The purpose in this article is to present different activities for each of these categories and to encourage mathematics teachers to try implementing these suggestions in their own instruction.

Direct Use of Language

This level of writing involves students in copying and recording information.

Although most mathematics teachers require that students take notes, notetaking is more effective if the students are given some structure. Many of the teachers checked their students' notebooks periodically, but the students viewed the notebook as more important when the checking was more frequent. In one classroom, the teacher would spot-check the students' notebooks daily and give the class advice on maintaining the notebooks as a useful study tool. Another teacher impressed on the students the necessity for completeness and accuracy in their notebooks by giving periodic quizzes in which the students had

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to use their notebooks to find the answers.

Linguistic Translation

Activities in this category call for the translation of mathematical symbols into written language.

One task in this category is the translation of symbolic expressions into words. The following is an example of this type:

Write in words: $(2n + 5)/3$

Here the student makes a straightforward translation from the symbolic meaning to the English language. Variations of this exercise would include the following:

Explain the meanings of the symbols in the formula $d = rt$.

Translate the formula $A = lw$ into words.

Another important task in this category is having students translate their solution to a word problem into a complete English sentence. For example, the solution may be $x = 9$, but students need to complete the problem by writing "The length of the garden plot is 9 meters."

Writing activities within this category do not necessarily have to be rigid exercises in translation. One teacher gave the class a long division problem:

$$23\overline{)276}$$

Each student wrote out the problem in English and then wrote out the steps to solve the problem. The teacher then conducted a whole-class discussion in which various students read their written responses aloud. These were written on the chalkboard. Then the teacher showed an overhead transparency of his writing of this problem. Students were allowed to compare their answers to that of the teacher. This modeling was important because it allowed the students to know what was expected of them.

As a follow-up exercise, each student repeated this writing process, working with an individually designed problem. This written problem was given to another student, who had to

Fig. 1 A student's efforts to describe a solution to $\frac{40}{100} = \frac{x}{75}$

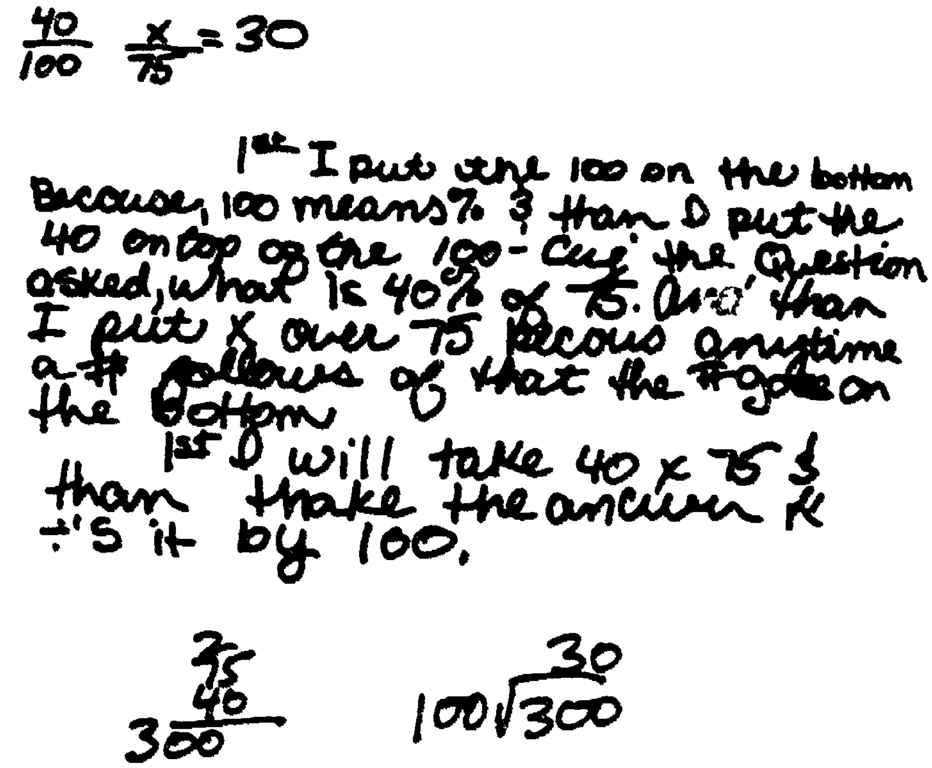
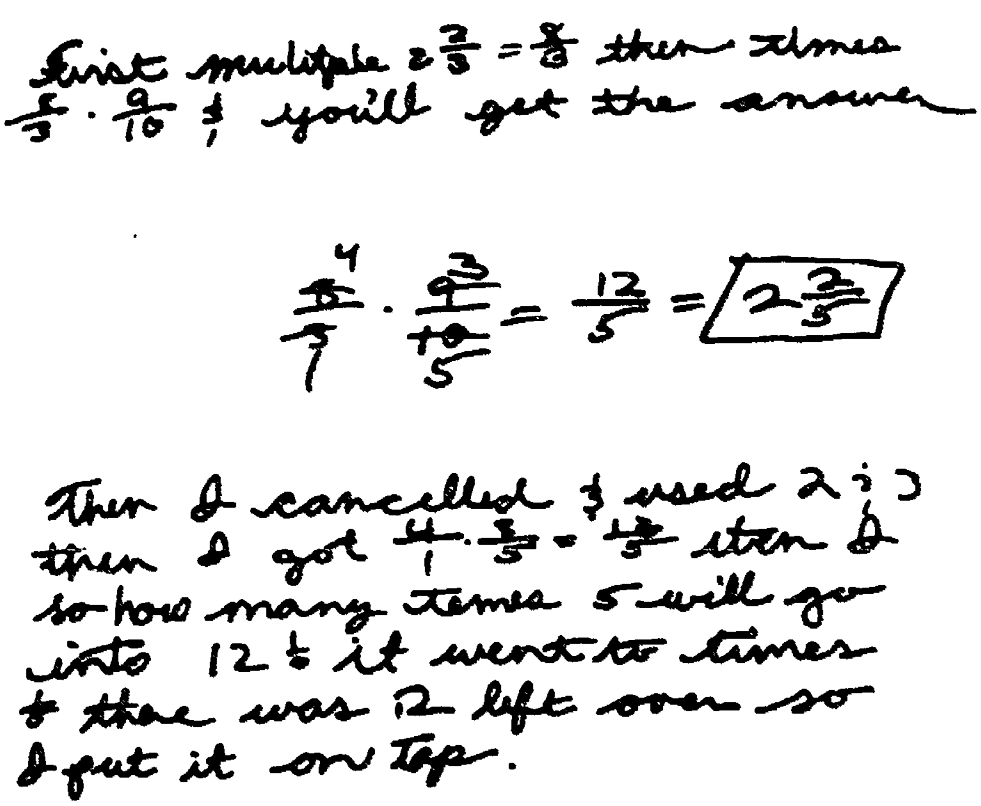


Fig. 2 A student's explanation of how to evaluate $2\frac{2}{3} \times \frac{9}{10}$.



reconstruct and solve the problem from the written sheet and give the results in mathematical symbols. The students found this activity enjoyable and approached it as a puzzle. This procedure varied the routine of giving students whole-class assignments.

Summarizing

Summarizing activities include paraphrasing or summarizing material from the textbook or some other source (such as classroom presentations).

Writing activities in this category can take a wide variety of guises. For example, rather than just present the material for transcription into notes, the teacher could discuss the material and then ask the students to record notes from memory. One extension of this type of activity is to have the students describe solutions in detail. One such example would be to have the students describe how they solve proportion problems (fig. 1). Another example, a student's description of how to multiply two fractions, is shown in figure 2. Writing of this type can either be incorporated into existing notebooks or be recorded in a special-purpose notebook.

Another summarizing activity is to have students keep a personal mathematics journal. A journal can consist of reactions to the mathematics being studied, such as questions about specific problems encountered and personal insights into the processes of mathematics. One such example is "Great! I found a new way to change decimals to fractions." When the teacher read the journal it showed her how successful the student was in understanding the material. Similarly, the journals of other students pointed up strengths and weaknesses in their comprehension of the material.

It should be noted this is the first writing application discussed thus far that is not intended specifically to improve mathematics performance. A journal is a communication device and certainly, when read by the teacher, serves as an instrument for both improving writing ability and communicating with the teacher. If attention is paid to the content of the journal, in contrast to mechanics of language (spelling, sentence fragments, punctuation), the teacher will learn how the student is interacting with the mathematics being studied.

This form of writing can help students learn more about mathematics in two ways. First, the act of writing about the material serves to clarify the mathematical process in the student's mind. Thus, students communicate how well they understand the content. A student may have misunderstandings that do not show up in homework assignments and tests. Such misconceptions can trouble the student who scores well in graded work but is unclear about some apparently minor detail. An example is the student who wrote, "I'm glad I got that question right on the test, but I'm not really sure what I'm doing." Because no warning signals are triggered by test results, such a student may believe the misconception to be unimportant. By examining the journal, the teacher can be alerted to potential or actual problem areas and take appropriate remedial action.

Summarizing activities involve the students in such important processes as considering and reflecting on mathematical concepts and using mathematical terms. To be most effective, however, students' summarizing efforts need to be read periodically by the teacher to insure accuracy of content and to provide feedback. However, this feedback does not always have to come from the teacher. It can also come from other students who read and comment on each other's work.

Applied Use of Language

The applied use of writing activities includes situations where the student applies a mathematical idea to a problem context.

In many classrooms, the students were required to make up story problems. In one instance, the teacher had the students cut out a picture from a newspaper or magazine and write a two-step story problem based on the picture (figure 3). In another instance, the class had just completed a unit on solving two-step linear equations in one variable. As an exercise, the students were given a set of equations and were asked to write related story problems. By way of follow-up, each student was assigned the task of making up another story problem that would lead to a two-step linear equation. After the problems were written, students exchanged problems with their neighbors, who solved the problems and returned them for correction. The teacher collected the original problems and their solution efforts for grading purposes.

Another teacher indicated that she asked each student to write one problem related to the topic just completed. From the problems submitted, she selected a set to give to the students as homework. The teacher also retained some questions for future use on a quiz or test.

A different approach used by another teacher was having students prepare quiz or test questions. The students turned in their questions and solutions as a homework exercise. The teacher then selected some of these questions for inclusion on the unit test.

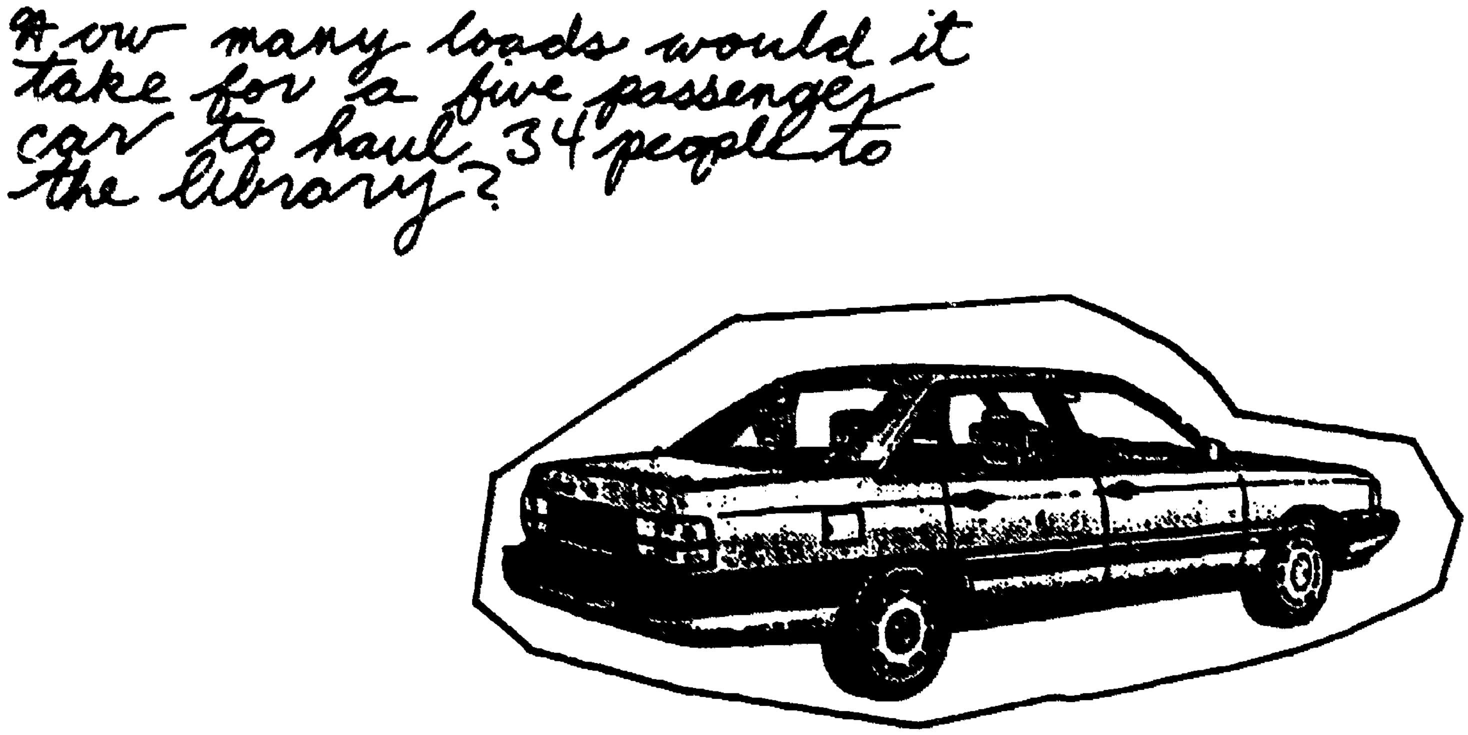
Activities of this type are important because the ability to write about mathematics shows that the student can apply the concepts learned to a real-world situation. In particular, having students write their own story problems is an effective way to have them analyze problem-solving strategies. For example, by exchanging problems with other students, each student gets practice in solving problems that are more realistic than typical textbook exercises. Students are also more inclined to make an honest effort to solve problems that are viewed as more realistic.

Creative Use of Language

In the category of creative use of language, the student uses written language to explore and convey information that, although it may contain mathematics concepts, is not specifically being studied in mathematics.

Potential assignments in this category typically are viewed as outside

Fig. 3 A picture of a car prompted a student to create this example.



the development of mathematical skills. Although many of the teachers we have worked with agreed that such assignments allow students the opportunity to apply mathematics skills to real-world situations, relatively few of them use such assignments. One teacher assigned the writing of short papers on important people and events in the history of mathematics. As an alternative, the students could make oral reports to the class and submit their speaking notes to the teacher. Sample topics included "ancient attempts to find the value of pi" and "Pythagoras's contribution to the study of numbers."

Research projects represent a significant means of enabling students to integrate writing and mathematical skills. More than one teacher used statistical projects in this way. Groups of students were assigned to investigate a topic and to write a report. One example was an investigation into whether students at different grade levels were more (or less) likely to watch situation comedies on television. The data-collection process gave students an opportunity to experience what happens in this type of project. In writing the report the students synthesized the findings of the research and communicated them in a coherent narrative. Later, the teacher posted the papers on a bulletin board for the class to read.

Summary

Having students engage in writing tasks in a mathematics context has several benefits. Not only will the opportunity to practice writing improve a student's ability in written expression, but using writing to practice mathematical tasks will also assist students in comprehending mathematics concepts and improve their ability to communicate mathematically. The inclusion of writing activities, such as those presented in this article, potentially has additional value when the kinds of activities are varied and writing is treated as an attempt at communication. In such an environment writing can become both a personal and a rewarding activity for students. Given that many students view mathematics as a stringent program of rules, facts, and figures, writing activities can involve students in useful and enjoyable mathematical activities. These, in turn, can encourage students to become more proficient in mathematics.

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Problem Solving
Tips for Teachers

AN AGENDA FOR ACTION
NCTM
1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998

Strategy Spotlight

Asking Questions to Evaluate Problem Solving

Sometimes we ask questions that are designed primarily to stimulate students to think. Other times questions are asked to give students help in solving a problem. But have you thought about how you could ask questions to help evaluate your students' thinking and attitudes about problem solving?

Checking a student's written work for a problem gives you important information about that student's ability to solve problems, but it does not provide much information about the student's thinking or how the student feels about problem solving. Evaluative questions of the following types can help fill this information gap:

- How did you. . . ?
- What did you try. . . ?
- Have you. . . ?
- How did you decide. . . ?
- Are you sure that. . . ?
- How do you feel about. . . ?
- Why did you. . . ?
- How do you know that. . . ?
- How did you happen to. . . ?
- Can you describe. . . ?
- What do you think. . . ?

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Consider the following problem:

> A frog is at the bottom of a well 10 meters deep and 2 meters across the top. On the first day he climbs up 5 meters but at night slips back 4 meters. If he does this each day, on what day will he get out of the well?

Here are some sample questions you might ask while a student is solving this problem or after a solution has been found.

- To help evaluate the student's ability to understand the problem, ask—
  "*Can you describe* this problem in your own words? What does 'get out of the well' mean?"
  "*Does this problem* remind you of any others you have solved before?"
  "*What did you do* to help you understand this problem?"
  "*How do you know that* 10 is not the correct answer?"
- To help evaluate the student's ability to deal with data, ask—
  "*How did you decide* which data are needed to solve this problem?"
  "*How do you know that* the depth of the well is needed to solve the problem?"
- To help evaluate the student's ability to choose appropriate strategies, ask—
  "*Can you suggest* some strategies that might help you solve this problem?"
- To help evaluate the student's ability to carry out a plan, ask—
  "*Have you* used strategies in solving the problem? Which ones?"
  "*What did you do* first to solve this problem?"
  "*Can you describe* your solution to the problem?"
- To help evaluate the student's ability to determine the reasonableness of an answer, ask—
  "*Are you sure* this is the answer to the problem? Why?"

"*Do you think* there might be other answers?"

- To help evaluate the student's attitudes and beliefs, ask—

  "*Do you like* to solve problems like this? Why or why not?"

  "*How do you feel* about your experience with this problem?"

  "*Do you think* there might be another way to solve this problem?"

As your students answer questions like these, you may wish to record your evaluations using a checklist like the one in figure 1. Remember, good questions will allow you to evaluate aspects of your students' performance and attitudes that are difficult, if not impossible, to evaluate using other evaluation techniques. The information gleaned from your evaluative questions can then be used to help you plan subsequent instruction.

Fig. 1
Problem-solving Question Response Checklist

Date ________ Name ________________

___ 1. Tries to understand the problem before computing

___ 2. Pays attention to data needed and not needed

___ 3. Thinks about which strategies might help

___ 4. Is flexible—tries different strategies if needed

___ 5. Understands the need for a plan

___ 6. Carries out a plan in a systematic manner

___ 7. Checks to see if an answer is reasonable

___ 8. Perseveres—sticks with a problem

___ 9. Demonstrates self-confidence

___ 10. Is able to describe or analyze a solution

Tip Board

Problem Corner

You may wish to ask evaluative questions as students in grades 6–8 solve this "golden" problem. It is a good problem for assessing a student's ability to describe and explain his or her solution.

> A seller had eight gold coins that he claimed had the same weight. The coin dealer who wanted to buy them suspected that one of the pieces was heavier than the other seven. She was right and found the heavier coin by using a balance scale and only two weighings. How do you think she did it?

(*Solution*: Place three coins on each balance. If they balance, the heavier coin can be found by trying to balance the other two coins. If they don't balance, try to balance two of the coins from the heavier set of three. If they balance, the third coin is the heavy coin. If they don't balance, the heavier coin is determined.—*Ed.*)

Classroom Climate

When you ask questions for evaluation purposes, remember these considerations:

- Ask the questions in a friendly, relaxed, nonthreatening manner.
- Reduce students' anxiety by discussing with them your use of questioning techniques for the purpose of assessment.
- Be sure students know that you are evaluating them to find ways to help them become better problem solvers and not for the purpose of assigning a grade.
- Share with students insights gleaned from your evaluative questions that they can use to improve their problem-solving skills.

Take a Look

For more information on questioning and evaluating problem solving, take a look at the new NCTM booklet *How to Evaluate Progress in Problem Solving* by Randall Charles, Frank Lester, and Phares O'Daffer (1987).

To see some other important roles of questioning, read the article "The Role of Questioning" by Marilyn Burns in the February 1985 *Arithmetic Teacher* (Focus Issue on Mathematical Thinking).

Part of the Tip Board is reserved for techniques that you've found useful in teaching problem solving in your class. Send your ideas to the editor of this section.

7. A Sisyphean Task: Historical Perspectives on the Relationship Between Writing and Reading Instruction. $3.50
G. Clifford (A joint report with the Center for the Study of Reading.)

Using perspectives drawn from American educational and social history, Clifford identifies five historical forces and probes their interacting influence on English language education: the democratization of schooling, the professionalization of educators, technological change, the functionalist or pragmatic character of American culture, and liberationist ideologies.

8. Writing and Reading in the Classroom. $3.00
J. Britton (A joint report with the Center for the Study of Reading.)

Britton explores the classroom as an environment for literacy and literacy learning. He discusses ways in which teachers have developed strategies for encouraging children to learn to write-and-read—activities that have often been dissociated in classrooms but that together create a literacy learning environment.

9. Individual Differences in Beginning Composing: An Orchestral Vision of Learning to Write. $3.00
A.H. Dyson

Looking in depth at three first graders during classroom journal time, Dyson explores the interconnections of the children's speaking, writing, and drawing as indications of their developing acquisition of written language. Her analysis reveals the complexity of the writing acquisition process, as the three symbol systems interact in different ways for the different students.

10. Movement Into Word Reading and Spelling. $3.00
L.C. Ehri (A joint report with the Center for the Study of Reading.)

Drawing on studies of the role of spelling in the reading process, Ehri discusses ways in which spelling contributes to the development of reading and, conversely, how reading contributes to spelling development. The role of writing in reading and spelling development is also discussed.

11. Punctuation and the Prosody of Written Language. $3.00
W. Chafe

Prosody—rises and falls in pitch, accents, pauses, rhythms, variations in voice quality—while a salient feature of spoken language, is not fully represented in written language. Reporting on a study of younger and older readers, Chafe explores the relationship between what he calls the covert prosody of writing and the principal device that writers use in order to make it at least partially overt, the devise of punctuation.

12. Peer Response Groups in Two Ninth-Grade Classrooms. $3.00
S.W. Freedman

Freedman looks at peer response groups in two ninth-grade college preparatory classrooms. Her analysis of the students' face-to-face interactions reveals how students approach the substance and form of their writing, self- and other-evaluation, problem-solving, and audience awareness.

13. Writing and Reading: The Transactional Theory. $3.00
L. Rosenblatt (A joint report with the Center for the Study of Reading.)

This report focuses on some epistemologically-based concepts relevant to the comparison of the reading and writing process which Rosenblatt believes merit fuller study and application in teaching and research.